# Module 4: Ratios

## RATIOS, RATES AND PROPORTIONS

A **ratio** compares the sizes of two quantities; the quantities may or may not have units.

A **rate** is a ratio of two quantities measured in different units. Rates are often a 'quantity per unit of time'.

A **proportion** describes the relationship of a part to the whole.

### Worked examples

1. In a laboratory class of students there are 16 males and 24 females. The ratio of males to females would be written as 16 : 24

   The total number of students in the class is 40 so the proportion or fractional share of males in the class is $\frac{16}{40}=\frac{2}{5}$ and the proportion of females in the class is $\frac{24}{40}=\frac{3}{5}$

   Note that the ratio of males to females is equivalent to the ratio of the proportion of males to the proportion of females:

   $\frac{16}{40}:\frac{24}{40}$ or 16 : 24 (multiply both sides by 40) or 2 : 3 (divide both sides by 8)

   They are all equivalent ratios. Note that the proportion of males is $\frac{2}{2+3}=\frac{2}{5}$ and the proportion of females is $\frac{3}{2+3}=\frac{3}{5}$

2. A beaker contains 250 ml of liquid A and 600ml of liquid B.

   The ratio of the amount of liquid A to the amount of liquid B is 250 : 600 which is equivalent to 50 : 120 (divide by 5) or 5 : 12 (divide by 10) .

   The proportion of liquid A in the beaker is $\frac{250}{850}=\frac{5}{17}$, and of liquid B is $\frac{600}{850}=\frac{12}{17}$

3. Heart rate is measured in beats per unit of time. Ben counted 21 heart beats in 15 second using his wrist pulse. How many times does Ben's heart beat every minute?
   One minute is the same as 60 seconds, and 60 ÷ 15 = 4 . To convert 15 seconds to 1 minute, multiply by 4. So to convert the rate to beats per minute, multiply both parts by 4. $\frac{21\text{ beats}}{15\text{ seconds}}=\frac{21\times4\text{ beats}}{15\times4\text{ seconds}}=\frac{84\text{ beats}}{60\text{ seconds}}=\frac{84\text{ beats}}{1\text{ minute}}=84\ \frac{\text{beats}}{\text{minute}}$

**Exercise 1**

1. Express the following ratios in simplest form:

(a) Length (cm) to width (cm) - a block of wood with length 5.5cm and width 3.5cm

(b) Flour to sugar (cups) - a recipe requires three quarters of a cup of flour and half a cup of sugar

2. A mineral assay of a sample shows that an ore is only made up of two metals, iron and nickel, in a ratio of 9 : 11.

(a) What proportion of the sample ore is iron?

(b) For an ore deposit mass 1800kg, what mass of nickel would you expect to be in the deposit based on this assay result?

3. There are 20 Merino sheep in a paddock and the rest are Poll Dorset sheep. If there are 100 sheep in total,

(a) What is the ratio of (i) Dorset to Merino sheep?

(ii) Merino to Poll Dorset sheep?

(b) What is the proportion of Poll Dorset sheep?

(c) What is the percentage of Merino sheep?

4. What proportion of 42 is (a) 6 (b) 8 (c) 14 ? Write your answer as a fraction in the simplest form.

5. If 75% of science students at Murdoch University are studying mathematics, what is the ratio of those studying maths to those not studying maths?

## EQUATING PROPORTIONS

### Worked examples

1. How much salt is needed to make up one litre of a saline solution with the same concentration as a saline solution containing 5 mg of salt in 100 ml of water?

   Let *x* be the number of mg of salt needed in the 1 litre of solution.

   So in the new one litre solution there will be x mg salt in 1000ml water

   We equate the proportions of salt and water in the two solutions $\frac{new\ solution}{given\ solution}$

   to give $\frac{x}{5} = \frac{1000}{100}$ so $\frac{x}{5} = 10$ and multiplying both sides by 5 gives $x = 50$.

   So I need 50 mg of salt to give the same salt concentration in 1 litre of liquid.

*2.* Twelve oranges cost $5. How much will 20 oranges cost?

   Let 20 oranges cost $*x*

   Equating the proportions of quantity and cost gives $\frac{20}{12} = \frac{x}{5}$ and $\frac{20}{12}(5) = x$

   So $x$ = 8.333... , so, to the nearest cent, the cost of 20 oranges is $8.33.

3. A large water tank takes 16 minutes to fill to 55 litres.
   (a) How long will it take to fill if the tank holds 200 litres?
   Let the x be the time taken for the tank to fill to 200 litres.
   Equate the proportions:

   $\frac{x}{16} = \frac{200}{55}$ ,

   $x = \frac{200}{55}(16)$

   so x = 58.1$\dot{8}$ ≈ 58 mins.

   (b) At what rate is water flowing into the tank (in litres per minute)?
   55 litres takes 16 minutes, so the flow rate of water into the tank is $\frac{55\ \text{litres}}{16\ \text{minute}}$ or $\frac{55}{16}$ litres per minute or 3.4 litres per minute. ( or use $\frac{200\ \text{litres}}{58\ \text{minute}}$, both give the same answer.)

**Exercise 2**

1. At Walter's cafe 5 salad rolls can be bought for $26.50.

   (a) How much would 7 salad rolls cost?

   (b) How many salad rolls could you buy for $58.30?

   (c) How many could you buy for $100?

2. A soft drink NEW is a mixture of liquid component A and liquid component B in the ratio of 1 : 4.

   (a) If you have 200ml of component A and an unlimited supply of component B what volume of the NEW drink can you make?

   (b) How much of component A is required to make 150 litres of the NEW soft drink?

3. A 750 ml copper sulphate solution was made by dissolving 200mg copper oxide in sulphuric acid (Assuming copper dioxide does not increase in volume).

   (a) How much copper oxide should be dissolved in 900ml of sulphuric acid to make the same concentration of copper sulphate solution?

   (b) To strengthen the concentration of copper sulphate solution by 25%, what would be the new ratio of copper oxide to sulphuric acid?

   (c) How much *extra* copper oxide should be added to the 900ml solution?

4. Methane is an important greenhouse gas that contributes to global warming. Every 16.04g of methane contains 12.01g of carbon. What mass of carbon would be found in 2 tonnes of methane?

## MORE WORD PROBLEMS

### Exercise 3

1. In an urban environment, a 2010 Mazda2 has a fuel consumption of 8.2 litres/100km and $CO_2$ emissions of 195 g/km. https://www.greenvehicleguide.gov.au/

(a) How far can a 2010 Mazda2 travel on a full tank (56 litres) of fuel?

(b) Given the average annual distance travelled in the Mazda2 is 14000km, calculate the average annual $CO_2$ emissions for the car. Give your answer in tonnes of $CO_2$

2. According to the Australian Bureau of Statistics (ABS) population clock, on the 23th November, 2015, the projected Australian population is 23,937,189. The projection is based on the estimated population at 31st March 2015 and assumes the growth rate since then to be;

"one birth every 1 minute 46 seconds"
"one death every 3 minutes 23 seconds"
"a net gain of one international migrant every 2 minutes 37 seconds"
“an overall total "population increase of 1 person every 1 min 32 secs"

http://www.abs.gov.au/AUSSTATS/abs@.nsf/Web+Pages/Population+Clock?opendocument

a) Convert the times to seconds and hence construct the three rates,

$$\frac{1 \text{ birth}}{? \text{ seconds}} \qquad \frac{1 \text{ death}}{? \text{ seconds}} \qquad \frac{1 \text{ migrant}}{? \text{ seconds}}$$

b) Use your calculator and the answers from part (a) to find the overall total population increase.

Overall population increase per second
= birth rate per second – death rate per second + migrant rate per second
$= \frac{1}{?} - \frac{1}{?} + \frac{1}{?}$
= (give your answer to 5 d.p.)

c) Does your calculation match that given by the ABS? Using the overall population rate from in part (b), approximately how long does it take to gain 1 person overall?

d) How many days would it take for the Australian population to grow by 100 000 people?

## ANSWERS TO EXERCISES

## RATIOS AND PROPORTIONS

### Exercise 1

1. (a) 11 : 7 (b) 3 : 2
2. (a) $^{9}/_{20}$ (b) 990 kg
3. (a) (i) 80 : 20 or 4 : 1 (ii) 1 : 4 (b) $^{80}/_{100}$ or $^{4}/_{5}$ (c) 20 %
4. (a) $^{1}/_{7}$ (b) $^{4}/_{21}$ (c) $^{1}/_{3}$
5. 75 : 25 or 3 : 1

## EQUATING PROPORTIONS

### Exercise 2

1. (a) $ 37.10 (b) 11 (c) 18
2. (a) 1 litre = 1000 ml (b) 30 litres
3. (a) 240 mg (b) 1 : 3 (c) 60mg
4. 1.50 tonnes carbon

## MORE WORD PROBLEMS

### Exercise 3

1. (a) $\frac{8.2\ L}{100\ km} = \frac{56\ L}{?km}$, $100 \times 56 \div 8.2 = 683\ km$

(b) $\frac{?\ g}{14000\ km} = \frac{195\ g}{1\ km}$, $14000 \times 195 \div 1 = 2{,}730{,}000 = 2.73\ tonne$

2.a)106 secs, 203 secs, 157 secs. The rates are:

b) The overall rate of population increase is;

$$\frac{1}{106} - \frac{1}{203} + \frac{1}{157} = 0.01088\ of\ a\ person\ per\ second$$

c) Yes! $\frac{0.01088\ Persons}{1\ sec} = \frac{\frac{0.01088}{0.01088}Persons}{\frac{1}{0.01088}Seconds} = \frac{1\ person}{92\ seconds} = \frac{1}{1\ \text{min}\ 32\ sec}$

c) 106 Days. $Note$: $1\ day = 24 \times 60 \times 60\ Sec$

$\frac{1\ Persons}{92\ sec} = \frac{100{,}000\ Persons}{9{,}200{,}000 Seconds} = \frac{100{,}000\ person}{153{,}333\ min} = \frac{100{,}000\ person}{2556\ hours} \frac{100{,}000\ person}{106\ days}$