Module 6: Equations and Formulas

There are many, equations and formulas which come from a wide range of disciplines and often provide a mathematical solution to real-life problems. In this module we will practice using basic algebra to solve and rearrange simple formulas.

SUBSTITUTING IN FORMULAS

Exercise 1

1. The formula for simple interest (SI) is SI=Prt, where P is the principal amount invested, r is the interest rate per annum and t is the time period in years. Using this formula, find the missing amounts in the table below (round to the nearest cent):

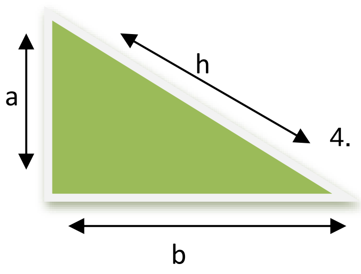
Principal	Simple interest ***rate*** per annum (convert percentages to decimal)	Time Principal amount is invested (convert to years)	Calculate simple interest	Calculate principal amount plus interest.
$800	4%	12 months	(a)	(b)
$3412	5.75%	9 months	(c)	(d)
$2100	6.4%	13 months	(e)	(f)

2. Use the following formula to find the value of z, when X = 102.3, μ = 87.7 and σ = 24.4.

$$z = \frac{X - \mu}{\sigma}$$

Use brackets around the numerator so that the numerator is calculated first.
i.e. (102.3 – 87.7) ÷ 24.4 =
OR
Press ''equals'' after typing in the numerator.
i.e. 102.3 – 87.7 = ÷ 24.4 =
Of these two computation methods, which do you prefer?

3. Use the Pythagorean equation, $h = \sqrt{a^2 + b^2}$, to find the length of the side h, in the right-angle triangle pictured left.



4. The length of side a = 3 and side b = 4.

5. Use the following formula to find the value of z when; $\bar{X}$ = 3.2, μ = 3.0, σ = 0.8 and n = 10.

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

NOTE: The numerator and denominator need to be calculated separately and then divided. One method is to use brackets around both numerator and denominator. i.e. (3.2-3)÷(.8÷√10)=

Operation	Inverse Operation	Numerical Examples
+	**–**	**2+7=9**
-	**+**	**2=9-7**
x	**÷**	**3x4=12**
÷	**x**	**3=12÷4**
Square	$\sqrt{\square}$	**8^2=64**
$\sqrt{\square}$	**Square**	**$\sqrt{64}$=8**

REARRANGING FORMULAS

Sometimes it is convenient to change the ‘’subject’’ of a formula. The formula V=kT gives the volume, V, of a fixed amount of gas at constant pressure, T is the temperature, and k is a constant. The subject of this formula is V.

Worked examples Let’s say, we would like to rearrange the formula, V = kT, so that we:

i. V=kT — Make k the subject of the formula.

$\frac{V}{T} = \frac{k\cancel{T}}{\cancel{T}}$ — Divide both sides of the equation by T.

The T’s on the right hand side (RHS) of the equation cancel.

$\frac{V}{T}=k$ or $\boxed{k = \frac{V}{T}}$

ii. V=kT — Make T the subject of the formula.

$\frac{V}{k} = \frac{\cancel{k}T}{\cancel{k}}$ — Divide both sides of the equation by k.

The k’s on the RHS of the equation cancel.

$\frac{V}{k}= T$ or $\boxed{T = \frac{V}{k}}$

iii. Let’s try a different formula, $V=\frac{k}{P}$ which gives the volume, V, of a fixed amount of gas at constant temperature, P is the pressure, and k is a constant.

$V=\frac{k}{P}$ — Make k the subject of the formula.

$V \times P = \frac{k}{\cancel{P}} \times \cancel{P}$ — Multiply both side of the equation by P.

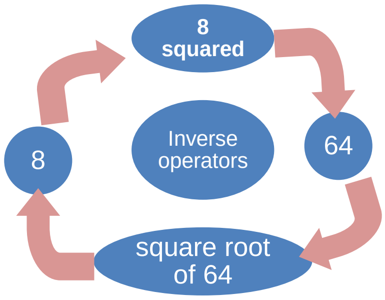
The P’s on the RHS of the equation cancel.

VP = k or $\boxed{k = VP}$

Exercise 2

1. Ohm's Law is, **V = IR** it describes the relationship between the voltage, V, the current, I, and the resistance, R, of an electric circuit. Using algebra rearrange the formula so that **I** is the subject of the formula.

2. Boyle's Law can be used to calculate changes in the volume or pressure of a fixed amount of gas at a constant temperature, the formula is: $\mathbf{V_1P_1=V_2P_2}$.
   Rearrange $V_1P_1=V_2P_2$ so that V_2, is the subject of the formula. (Note:V_1, P_1, V_2 and P_2 are four single variables.)



3. Rearrange the following formula so that C is the subject of the formula, that is in the form C = ...

$$A = \sqrt{B^2 + C}$$

Step 1: To ''reverse'' the square root sign, square both sides of the equation.

Step 2: Next, subtract B^2 from both sides of the equation, to get C by itself.

4. Rearrange the following formula so that u is the subject of the formula, that is u = ?

$$v^2 = u^2 + 2as$$

Step 1: Subtract 2as from both sides of the equation, to get u^2 by itself.

Step 2: To ''reverse'' u^2 into u; take the square root of both sides of the equation.

SOLVING EQUATIONS

Exercise 3

1. The following formula is called the 'ideal-gas equation'; it has 4 variables n, P, V and T and 1 constant R. **PV=nRT** For the given values of 3 variables, find the value of the 4th, in terms of the constant R.
   a. V=5, n=1, T=200
   b. P=2, n=3, T=250
   c. P=3, V=6, n=5
   d. P=1.5, V=4, T=270
2. The following formula has 5 variables v, d_1, d_2, t_1, t_2: $v=\frac{d_2-d_1}{t_2-t_1}$

   For each question below, calculate the value of the unknown variable, by substituting the known quantities into the formula and then rearranging the formula.

	v	d_1,	d_2	t_1	t_2
(i)	?	20	40	0	1
(ii)	60	10	?	0.5	1.5
(iii)	40	?	80	1	2.5
(iv)	50	0	75	?	2
(v)	90	20	200	0.25	?

WORD PROBLEM

Exercise 4

1. A box is to be constructed so that it has a square base and a volume of 1 cubic metre. If the height of the box is 1.2m, what is the size of the side of the base?
   a. Sketch a 3D picture of the box, label the height, h, and the unknown sides of the base, x.
   b. Volume of a box = area of base x height.
      Let V, represent the volume of the box. Write down the formula for the volume of this box.
   c. Check the units are the same, and then substitute the known values for V and h. Solve the equation for x. Give your answer to the nearest centimetre.

ANSWERS TO EXERCISES

SUBSTITUTING IN FORMULAS

Exercise 1

1. (a)\$32 (b)\$832 (c) \$147.14
   (d)\$3559.14 (e)\$145.60
   (f)\$2245.60
2. 0.60
3. 5
4. a) 0.8

REARRANGING FORMULAS

Exercise 2

1. $I = \frac{V}{R}$
2. $V_2 = \frac{V_1 P_1}{P_2}$
3. $C = A^2 - B^2$
4. $u = \sqrt{v^2 - 2as}$

SOLVING EQUATIONS

Exercise 3

1.

a. $5P = 200R$

$P = 40R$

b. $2V = 750R$

$V = 375R$

c. $18 = 5RT$

$T = \frac{18}{5R}$

d. $6 = nR270$

$n = \frac{6}{270R} = \frac{1}{45R}$

2.

i. $v = \frac{40-20}{1-0} = \frac{20}{1} = 20$

ii. $60 = \frac{d_2-10}{1.5-0.5} = \frac{d_2-10}{1}$

$60 = d_2 - 10$

$d_2 = 70$

iii. $40 = \frac{80-d_1}{2.5-1} = \frac{80-d_1}{1.5}$

$40 \times 1.5 = 80 - d_1$

$60 = 80 - d_1$

$d_1 = 20$

iv. $50 = \frac{75-0}{2-t_1}$

$50 \times (2-t_1) = \frac{75}{\cancel{(2-t_1)}} \times \cancel{(2-t_1)}$

$\frac{50(2-t_1)}{50} = \frac{75}{50}$

$2 - t_1 = \frac{75}{50}$

$t_1 = 0.5$

v. $90 = \frac{200-20}{t_2-0.25}$

$90 = \frac{180}{t_2-0.25}$

$90 \times (t_2-0.25) = \frac{180}{\cancel{(t_2-0.25)}} \times \cancel{(t_2-0.25)}$

$90 \times (t_2-0.25) = 180$

$\frac{\cancel{90}(t_2 - 0.25)}{\cancel{90}} = \frac{180}{90}$

$t_2 - 0.25 = 2$

$t_2 = 2.25$

WORD PROBLEM

Exercise 4

b. Volume of a rectangular prism = length x width x height

$V=x^2h$

c. $1=x^2 1.2$

$1=1.2x^2$ (Divide both sides by 1.2)

$$\frac{1}{1.2}=\frac{\cancel{1.2}x^2}{\cancel{1.2}}$$

$x^2=\frac{1}{1.2}$ (Take the square root of both sides)

$x=\sqrt{\frac{1}{1.2}}=\sqrt{0.8\dot{3}}\approx 0.91287m$

The side of the square base is 91 cm in length.